Diffraction due to a single slit

B.Sc. (Physics Hons.) Part-2

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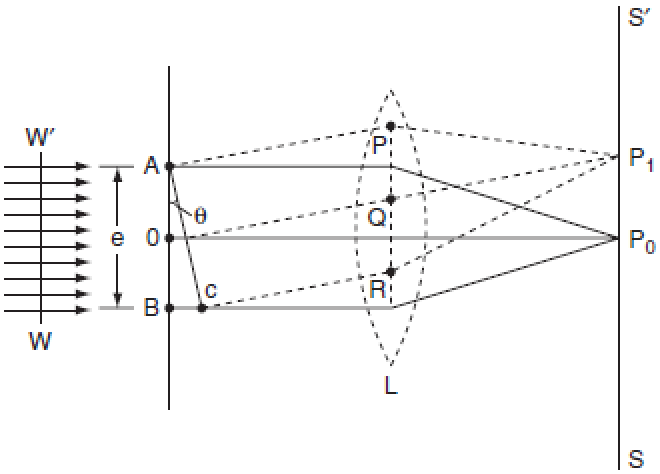
Diffraction due to a single slit:

The adjacent figure represents a narrow slit *AB* of width *'e'*. Let a plane wavefront of monochromatic light of wavelength 'λ' is incident on the slit. Let the diffracted light be focused by means of a convex lens on a screen. According to Huygen Fresnel, every point of the wavefront in the plane of the slit is a source of secondary wavelets. The secondary wavelets traveling normally to the slit *i.e., along* OP_o are brought to focus at P_o by the lens. Thus P_o is a bright central image. The secondary wavelets traveling at an angle 'θ' are focused at a point P_1 on the screen.

The intensity at the point P_1 is either minimum or maximum and depends upon the path difference between the secondary waves originating from the corresponding points of the wavefront.



In order to find out the intensity at P_1, draw a perpendicular *AC* on *BR*.

The path difference between secondary wavelets from *A* and *B* in direction q is *BC i.e.* ,

$$\Delta = BC = ABsin\boldsymbol{\theta} = esin\boldsymbol{\theta}$$

So, the phase difference,

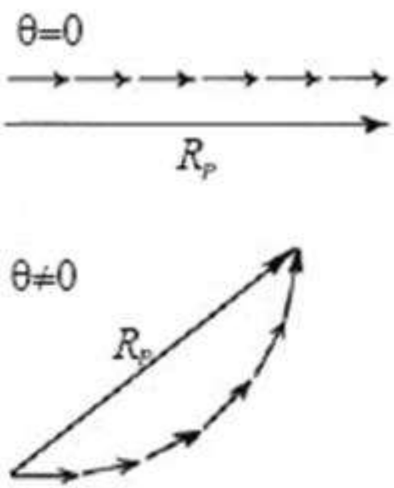
$$= \frac{2\pi}{\lambda} \times \Delta = \frac{2\pi}{\lambda}(esin\boldsymbol{\theta})$$

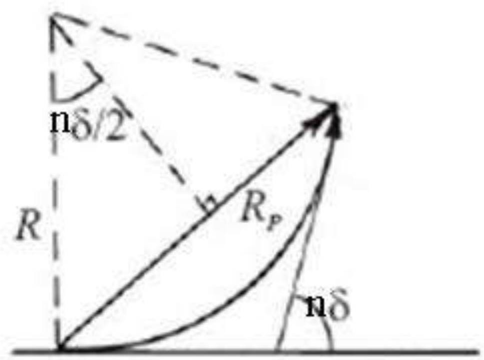
Let us consider that the width of the slit is divided into '*n*' equal parts and the amplitude of the wave from each part is *'a'*.

So, the phase difference between two consecutive points

$$\delta = \frac{1}{n}.\left\{\frac{2\pi}{\lambda}(esin\boldsymbol{\theta})\right\}$$ ....(1)

Then the *resultant amplitude R* is calculated by using the method of vector addition of amplitudes





The resultant amplitude of n number of waves having same amplitude 'a' and having common phase difference of 'δ' is

$$R = a\frac{sin(n\delta/2)}{sin(\delta/2)}$$ ....(2)

Substituting the value of δ in equation (2)

And Substituting $\alpha = \frac{\pi}{\lambda}.esin\boldsymbol{\theta}$

$$R = a\frac{sin\alpha}{sin(\alpha/n)}$$

As α/n is small value; $sin\alpha/n \rightarrow \alpha/n$

$R = na\frac{sin\alpha}{\alpha}$ and na = A

Therefore

$$\boldsymbol{R = A\frac{sin\alpha}{\alpha}} \text{ ....(3)}$$

Therefore, the Intensity is given by

$$I^2 = R^2 = \boldsymbol{A^2\frac{sin^2\alpha}{\alpha^2}} \text{ ....(4)}$$

<u>Case (i): *Principal Maximum:*</u>

Eqn (4) takes maximum value for

α= 0

$$\alpha = \frac{\pi}{\lambda}esin\theta = 0$$

$\Rightarrow sin\theta = 0$ or $\theta = 0$

The condition

The condition $\theta = 0$ means that this maximum is formed by the secondary wavelets which travel normally to the slit along OP_o and focus at P_o. This maximum is known as *"Principal maximum"*.

Intensity of Principal maxima

$$R_{max} = Lim_{\alpha\to 0}\frac{Asin\alpha}{\alpha} = ALim_{\alpha\to 0}\frac{sin\alpha}{\alpha}$$

$$R_{max} = A.1 = A$$

Therefore

$$I_{max}^2 = R_{max}^2 = A^2$$

Case (ii): *Minimum Intensity positions:*

Eqn (4) takes minimum values for $sin\alpha = 0$. The values of 'α ' which satisfy $sin\alpha = 0$ are

$$\alpha = \pm\pi, \pm 2\pi, \pm 3\pi .... \pm n\pi$$

$$\frac{\pi}{\lambda}.esin\theta = \pm n\pi$$

$\boldsymbol{esin\theta = \pm n\lambda}$ where $n = 1, 2, 3, ...$...(5)

in the above eqn (5) $n = 0$ is not applicable because corresponds to principal maximum. Therefore, the positions according to eqn (5) are on either side of the principal maximum.

Case (iii): *Secondary maximum:*

In addition to principal maximum at $\alpha = 0$, there are weak secondary maxima between minima positions. The positions of these weak secondary maxima can be obtained with the rule of finding maxima and minima of a given function in calculus. So, differentiating eqn(5) and equating to zero, we have

$$\because A^2 \neq 0; sin\alpha \neq 0$$

Because $sin\alpha = 0$ correspond to minima positions

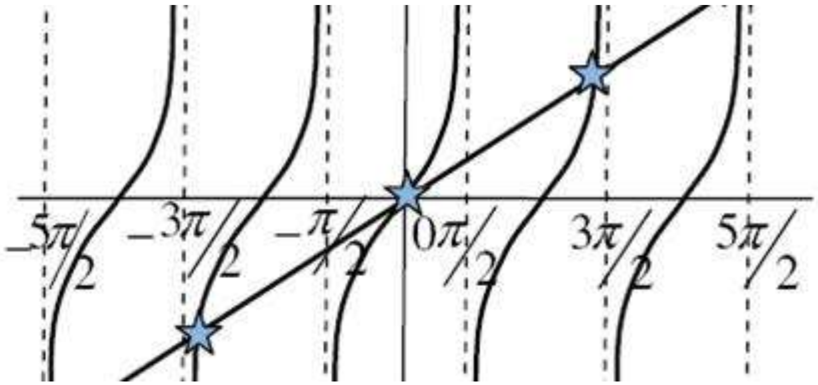
$\therefore \alpha coc\alpha - sin\alpha = 0$

$\Rightarrow \alpha = tan\alpha$...........(6)

The values of 'α' satisfying the eqn (6) are obtained graphically by plotting the curves $y = \alpha$ and $y = tan\alpha$ on the same graph. The points of intersection of the two curves gives the values of 'α ' which satisfy eqn (6).

The points of intersections are

$$\alpha = 0, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2} .... \pm \frac{(2n+1)\pi}{2}$$



But $\alpha = 0$, gives principal maximum, substituting the values of 'α ' in eqn(4), we get

$$I_1 = A^2 \left[\frac{sin3\pi/2}{3\pi/2}\right]^2 = \frac{A^2}{22}$$

$$I_2 = A^2 \left[\frac{sin5\pi/2}{5\pi/2}\right]^2 = \frac{A^2}{62}$$

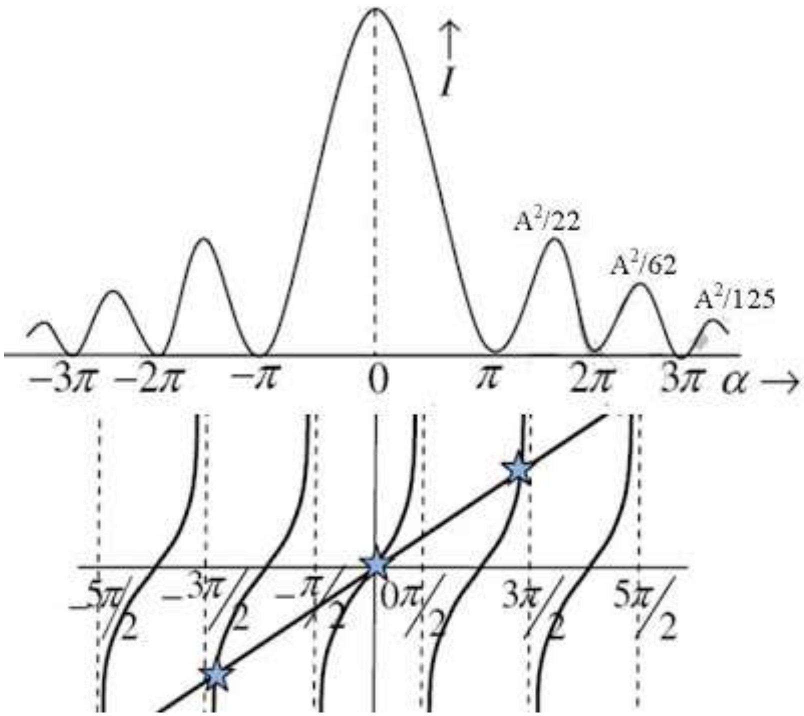
$$I_3 = A^2 \left[\frac{sin7\pi/2}{7\pi/2}\right]^2 = \frac{A^2}{125}$$

and so on.

From the above expressions, I_{max}, I_1, I_2,I_3... it is evident that most of the incident light is concentrated at the principal maximum.

Intensity distribution graph:

A graph showing the variation of intensity with 'α' is as shown in the adjacent figure



I
A²/22
A²/62
A²/125
−3π −2π −π 0 π 2π 3π α →
−5π/2
−3π/2
−π/2
0
π/2
3π/2
5π/2